

Statistics

Lecture 12



Feb 19-8:47 AM

Data {
1) Qualitative
2) Quantitative

{
1) Discrete
"Countable" SG 14-17
2) Continuous
"Measurable" SG 18-21

SG 18-19

Feb 1-4:34 PM

Working with Continuous random variable with Prob. dist

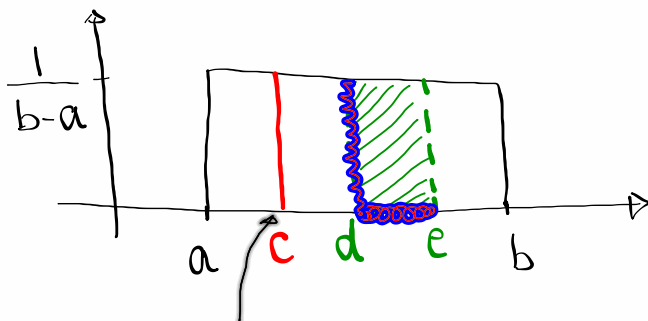
- 1) Uniform Prob. dist
- 2) Standard normal Prob. dist.
- 3) Normal Prob. dist.
- 4) Central limit theorem
- 5) Applications

SG 18-21

Feb 1-4:37 PM

Uniform Prob. dist.:

Let x be a continuous random variable for all values from a to b with a uniform Prob. dist.



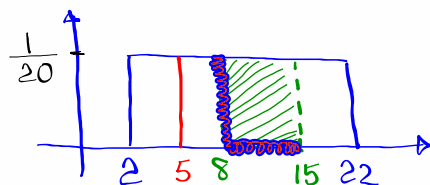
$$P(x=c)=0$$

$$P(d < x < e) = (e-d) \cdot \frac{1}{b-a}$$

Line has a Zero Area.

Feb 1-4:39 PM

Let x be a cont. random variable for all values from 2 to 22 with a uniform Prob. dist.



$$1) P(x=5) = 0$$

$$2) P(8 < x < 15)$$

$$= (15-8) \cdot \frac{1}{20}$$

$$= \frac{7}{20} = .35$$

3) Find $x = P_{90}$

90% below 10% above

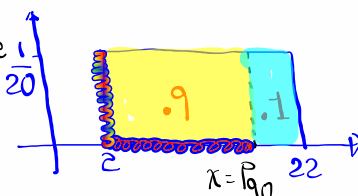
$$(x-2) \cdot \frac{1}{20} = .9$$

$$x-2 = 20(.9)$$

$$x-2 = 18$$

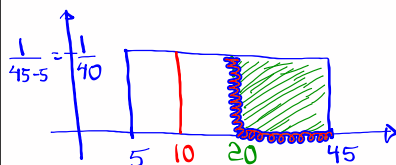
$$x = 20$$

$$P_{90} = 20$$



Feb 1-4:44 PM

Consider a uniform Prob. dist. for all values from 5 to 45. \rightarrow Rectangular graph



$$1) P(x=10) = 0$$

$$2) P(x > 20)$$

$$(45-20) \cdot \frac{1}{40}$$

$$= \frac{25}{40} = \frac{5}{8} = .625$$

3) Find two values that separate the middle 90% from the rest.

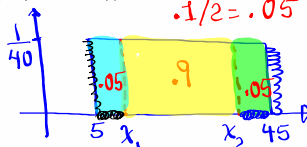
$$1 - .9 = .1$$

$$.1/2 = .05$$

$$(x_1-5) \cdot \frac{1}{40} = .05$$

$$x_1-5 = 40(.05)$$

$$x_1-5 = 2 \quad \boxed{x_1=7}$$



$$(45-x_2) \cdot \frac{1}{40} = .05$$

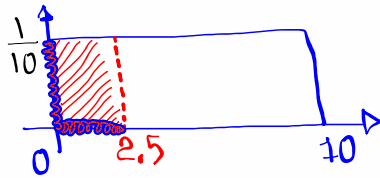
$$45-x_2 = 40(.05)$$

$$45-x_2 = 2$$

$$45-2 = x_2 \quad \boxed{x_2=43}$$

Feb 1-4:51 PM

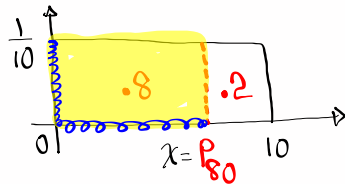
Habit Burger says wait time to be served has a uniform Prob. dist and it takes no more than 10 minutes.



Find the prob. for a randomly selected customer's wait time is below 2.5 minutes

$$P(X < 2.5) = (2.5 - 0) \cdot \frac{1}{10} = \frac{2.5}{10} = \boxed{\frac{1}{4}} = \boxed{.25}$$

Find the P_{80} of the wait-time



$$(x - 0) \cdot \frac{1}{10} = .8 \quad x \cdot \frac{1}{10} = .8 \quad x = 10(.8) \quad \boxed{x = 8}$$

SG 18 (Page 1-2)

Feb 1-5:01 PM

Standard Normal Prob. dist.:

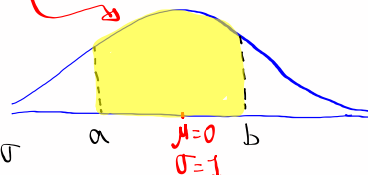
- 1) use Z , $P(Z = c) = 0$
- 2) Data dist. is symmetric, bell-shape with total area 1.
- 3) Mean = Mode = Median
- 4) $\mu = 0$, $\sigma = 1$
- 5) $P(a < Z < b)$ is the area of the corresponding region within the bell-shape.

How to find it:

`2nd` `VARS`

`normalcdf`

Lower, upper, μ , σ

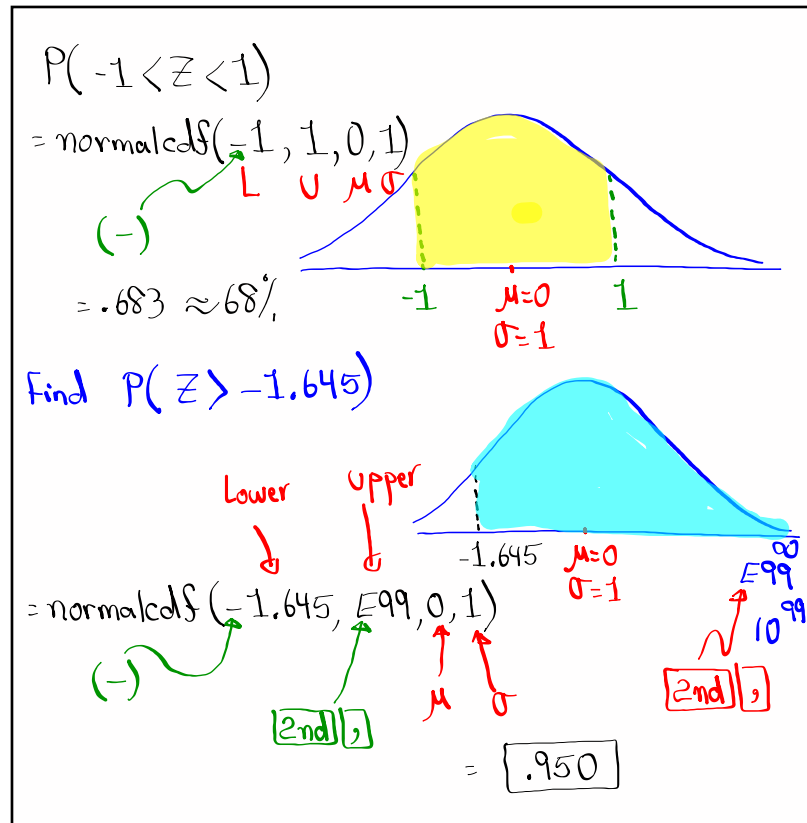


NO Exception

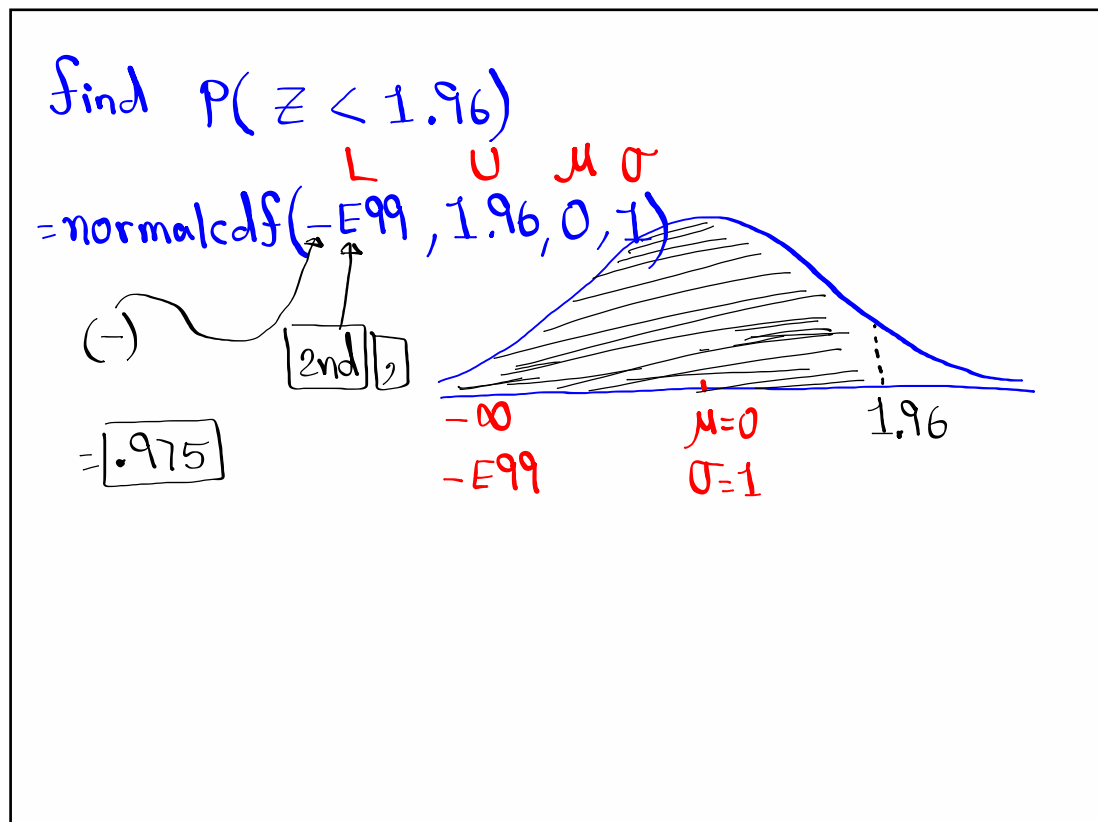
Full

Drawing, labeling, Shading, TI command Required

Feb 1-5:10 PM



Feb 1-5:17 PM



Feb 1-5:25 PM

Find $P(Z < -1.5 \text{ OR } Z > 1.75)$

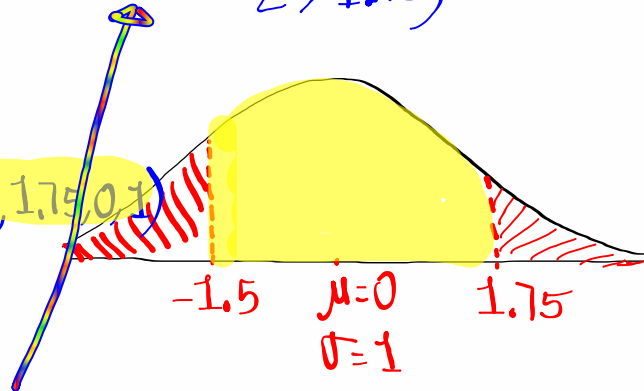
Total Area = 1

$$= 1 - \text{normalcdf}(-1.5, 1.75, 0, 1)$$

$$= \boxed{.107}$$

If OR was and

No Overlap $\rightarrow \boxed{0}$



Feb 1-5:27 PM

Find $Z = Q_3$

Third Quartile

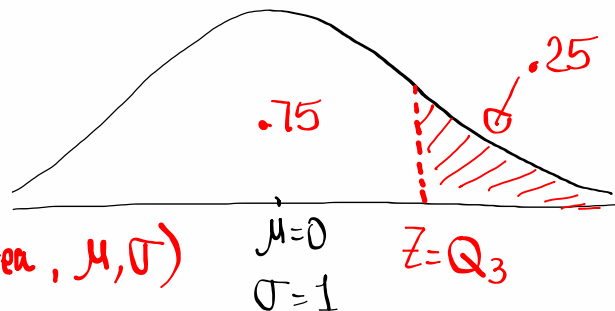
75% below, 25% above

Reverse Process

2nd

VARS

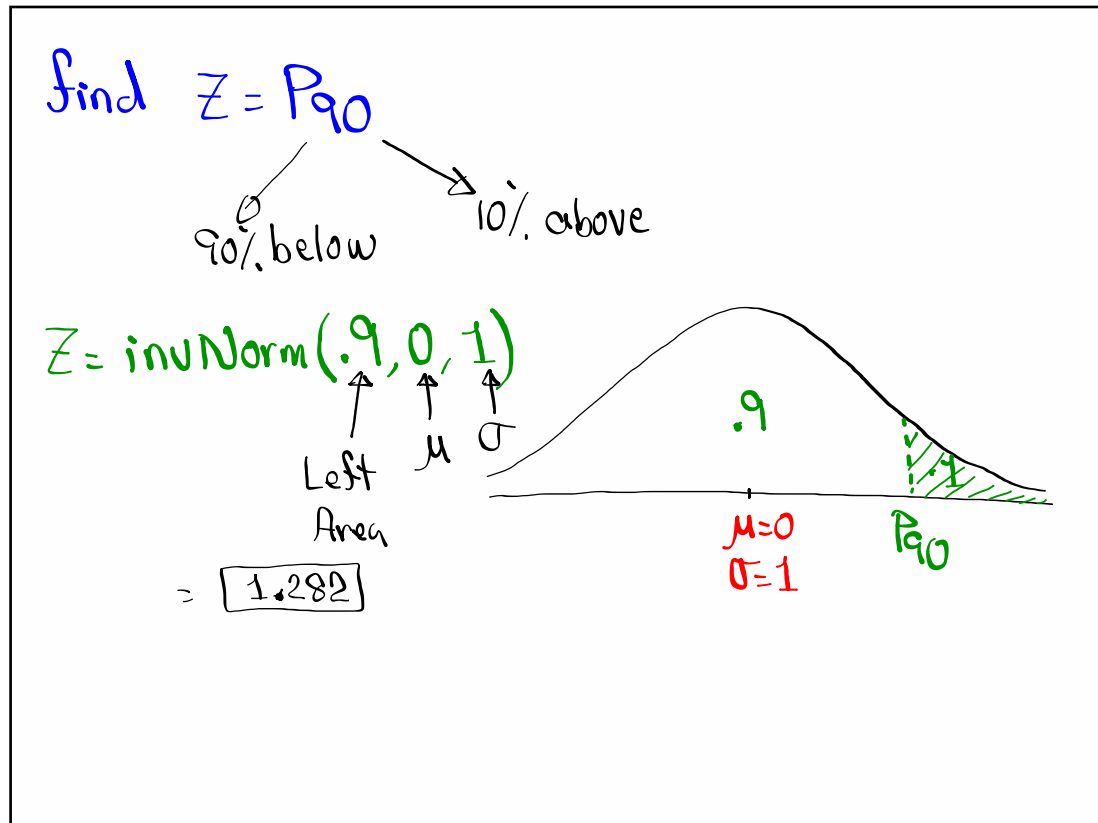
invNorm(Left Area, μ , σ)



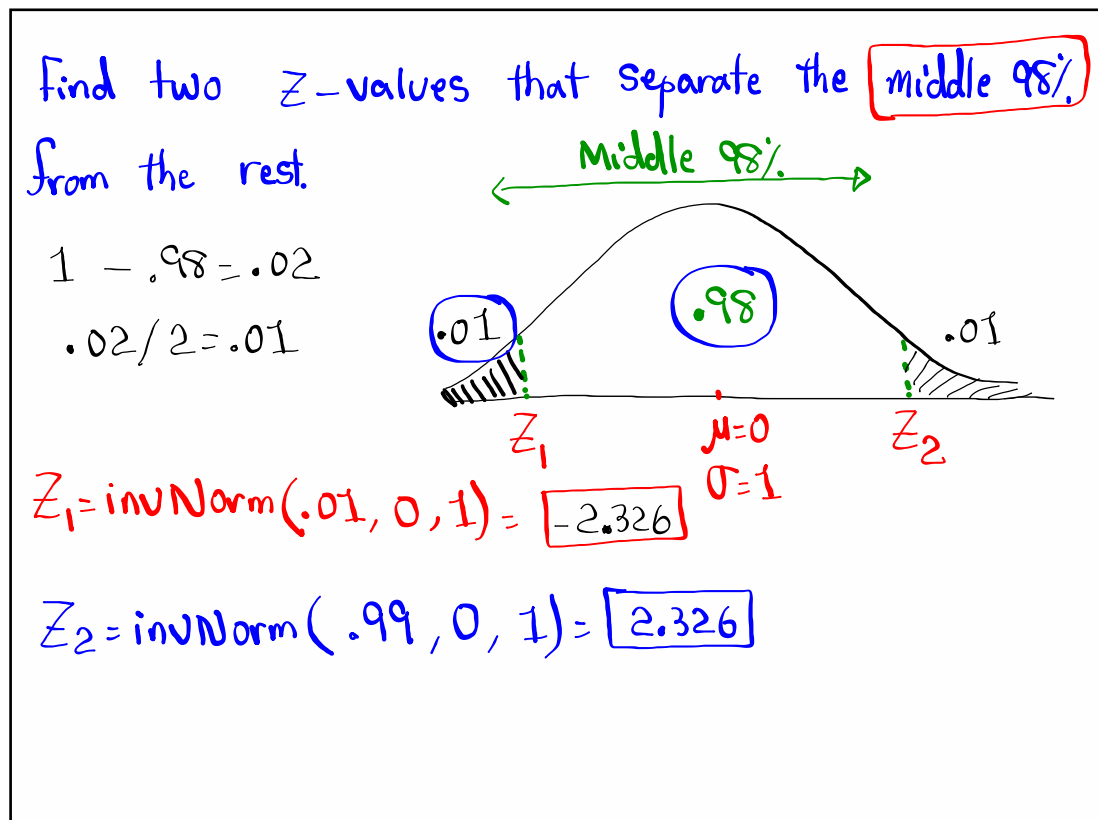
$$Z = \text{invNorm}(.75, 0, 1)$$

$$= \boxed{.674}$$

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Feb 1-5:37 PM



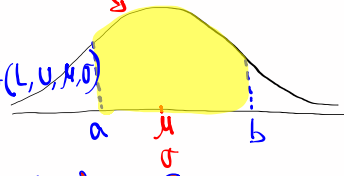
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Normal Prob. Dist.:

- 1) use X , $P(X=c) = 0$
- 2) Data dist is symmetric, bell-shape with total area 1.
- 3) Mean = Mode = Median
- 4) μ & σ are given in the Problem.
- 5) $P(a < x < b)$ is the corresponding area within the bell-shape graph.

once again,

we use $\text{normalcdf}(L, U, \mu, \sigma)$



Drawing, labeling, shading, full TI command required.

$N(\mu, \sigma)$

Feb 1-6:02 PM

Given $N(72, 8)$

Normal

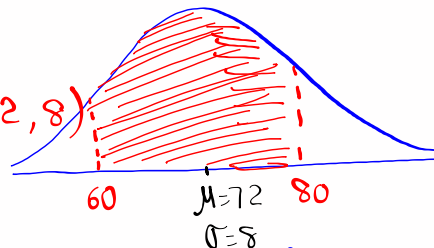
$\mu = 72$

$\sigma = 8$

$P(60 < x < 80)$

$= \text{normalcdf}(60, 80, 72, 8)$

$= .975$

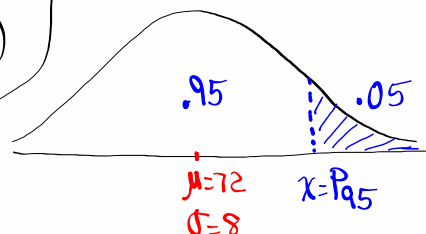


Find $x = P_{.95}$, Round-up your Final Ans.

$x = \text{invNorm}(.95, 72, 8)$

$= 85.1 \dots$

≈ 86



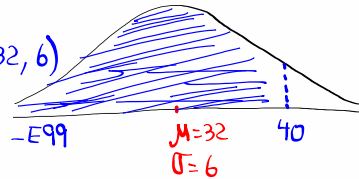
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Consider a normal Prob. dist with $\mu = 32$
and $\sigma = 6$. $N(32, 6)$

1) $P(x < 40)$

$= \text{normalcdf}(-E99, 40, 32, 6)$

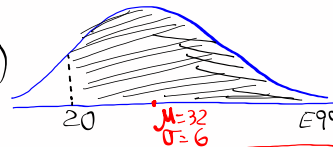
$= \boxed{.909}$



2) $P(x > 20)$

$= \text{normalcdf}(20, E99, 32, 6)$

$= \boxed{.977} \approx 97.7\%$

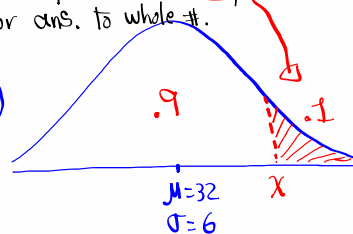


3) Find x -value that separates the top 10% from the rest. Round Your ans. to whole #.

$x = \text{invNorm}(.9, 32, 6)$

$= 39.689$

$\approx \boxed{40}$



Feb 1-6:14 PM

Salaries of nurses has a normal dist with
mean of \$6500/mo. and standard deviation
of \$400/mo. $N(6500, 400)$

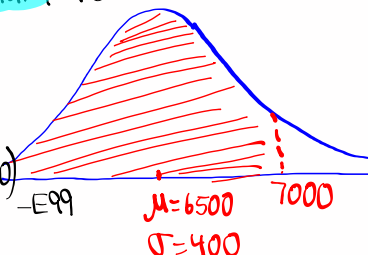
If we randomly select one nurse, find the
Prob. that his/her salary is

a) below \$7000/mo.

$P(x < 7000)$

$= \text{normalcdf}(-E99, 7000, 6500, 400)$

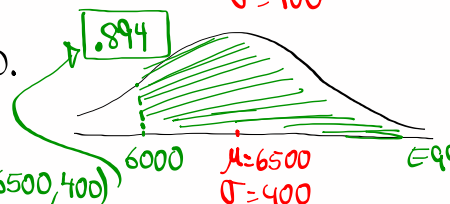
$= \boxed{.894}$



b) more than \$6000.

$P(x > 6000)$

$= \text{normalcdf}(6000, E99, 6500, 400)$



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Find Q_1 for the Salaries of nurses.

First Quartile

25% below,

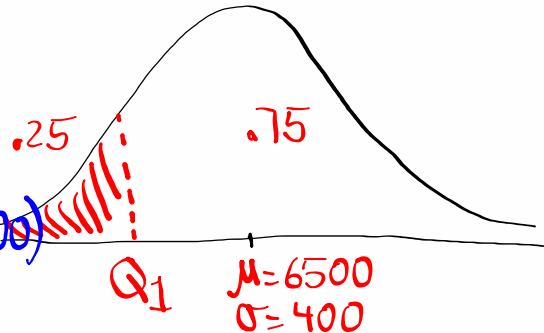
75% above

$$Q_1 = \text{invNorm}(.25, 6500, 400)$$

$$= 6230.2041$$

$$\approx \boxed{6230}$$

$$\$6230$$



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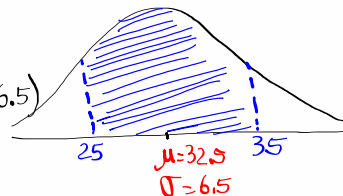
Ages of students are normally dist. with $\mu = 32.5$ and $\sigma = 6.5$ Yrs. $N(32.5, 6.5)$

If we randomly select one student,
Find the prob. that his/her age falls
between 25 & 35 Yrs.

$$P(25 < x < 35)$$

$$= \text{normalcdf}(25, 35, 32.5, 6.5)$$

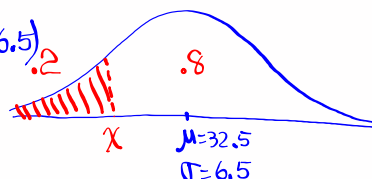
$$= \boxed{.525}$$



Find the age, rounded to 1-decimal, that
separates the bottom 20% from the rest.

$$x = \text{invNorm}(.2, 32.5, 6.5)$$

$$\approx \boxed{27.0}$$

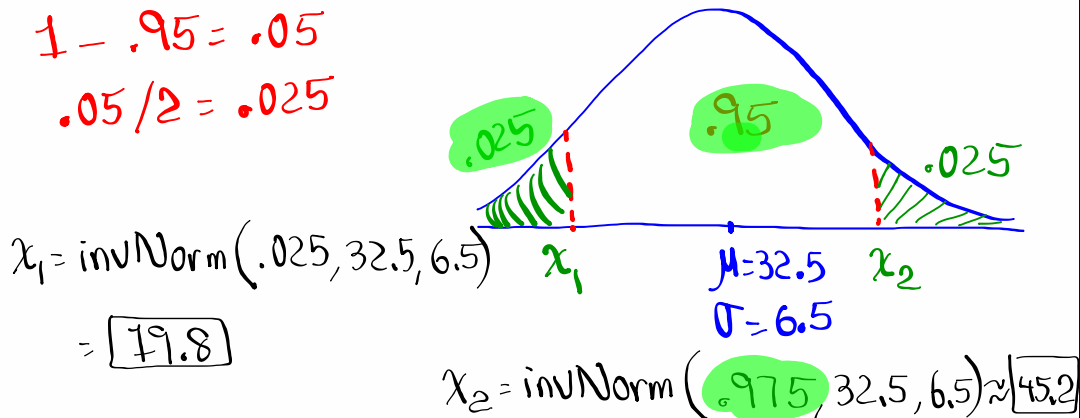


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Find two ages, rounded to 1-decimal, that
Separate the middle 95% from the rest.

$$1 - .95 = .05$$

$$.05 / 2 = .025$$



Feb 1-6:45 PM

You are making random guesses on a multiple-choice exam with 90 questions.

Each question has 6 choices but one correct choice.

Success is to guess correct answer.

$$n = 90 \quad p = \frac{1}{6} \quad q = \frac{5}{6}$$

$$\mu = np = 90 \left(\frac{1}{6} \right) = \boxed{15} \quad \sigma^2 = npq = 90 \left(\frac{1}{6} \right) \left(\frac{5}{6} \right) = \boxed{12.5}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{12.5} \approx 3.536 \approx \boxed{3.5}$$

Usual Range $\mu \pm 2\sigma = 15 \pm 2(3.5)$
 "95% Range" $= 15 \pm 7 \rightarrow \boxed{8 \text{ to } 22}$

Find the prob. of guessing at most one-fifth of them correctly.

$$\frac{1}{5} \cdot 90 = 18$$

$$P(x \leq 18) = \text{binomcdf}(90, 1/6, 18) = \boxed{.839}$$

Find the prob. of guessing at least one-tenth of them correctly.

$$\frac{1}{10} \cdot 90 = 9$$

$$P(x \geq 9) = 1 - P(x \leq 8) = 1 - \text{binomcdf}(90, 1/6, 8) = \boxed{.974}$$

Find the Prob. of guessing from 8 to 22 Correct ans.

$$P(8 \leq x \leq 22) = P(x \leq 22) - P(x \leq 7)$$

$$= \text{binomcdf}(90, 1/6, 22) - \text{binomcdf}(90, 1/6, 7) = \boxed{.967}$$

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